

QCD Thermodynamics at High Temperature

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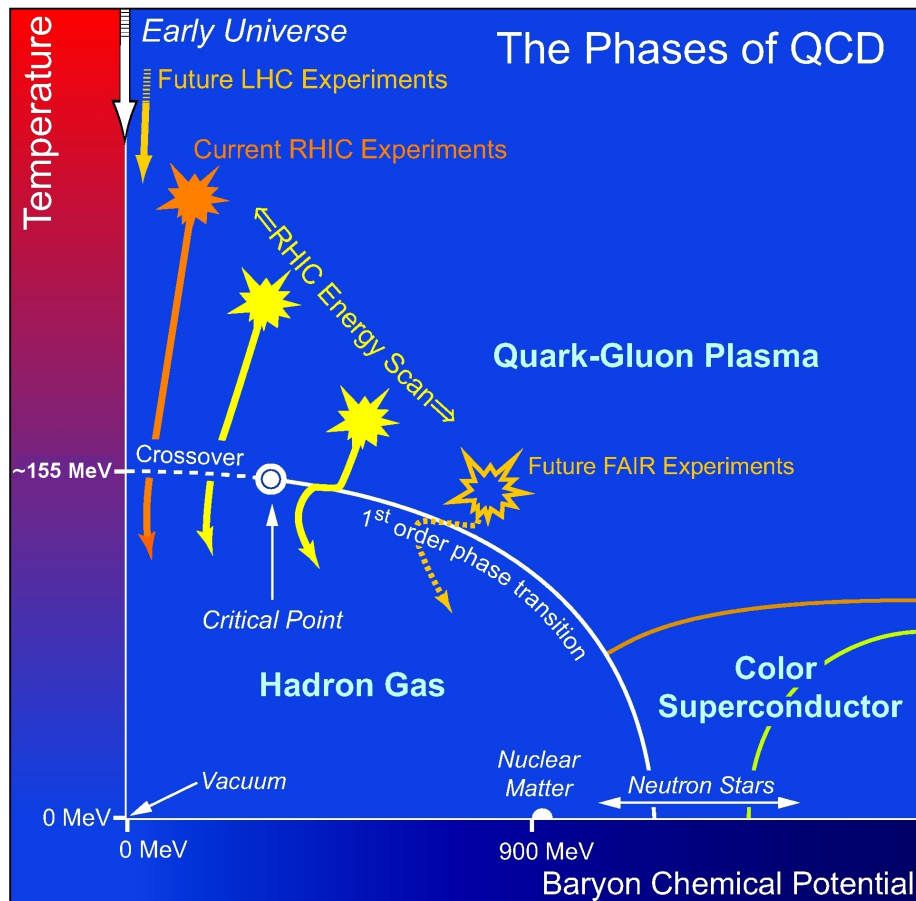
Large Scale Computing and Storage Requirements for Nuclear Physics (NP).
Bethesda MD, April 29-30, 2014

Defining questions of nuclear physics research in US:

Nuclear Science Advisory Committee (NSAC) “The Frontiers of Nuclear Science”,
2007 Long Range Plan

“What are the phases of strongly interacting matter and what roles do they play in the cosmos ?”

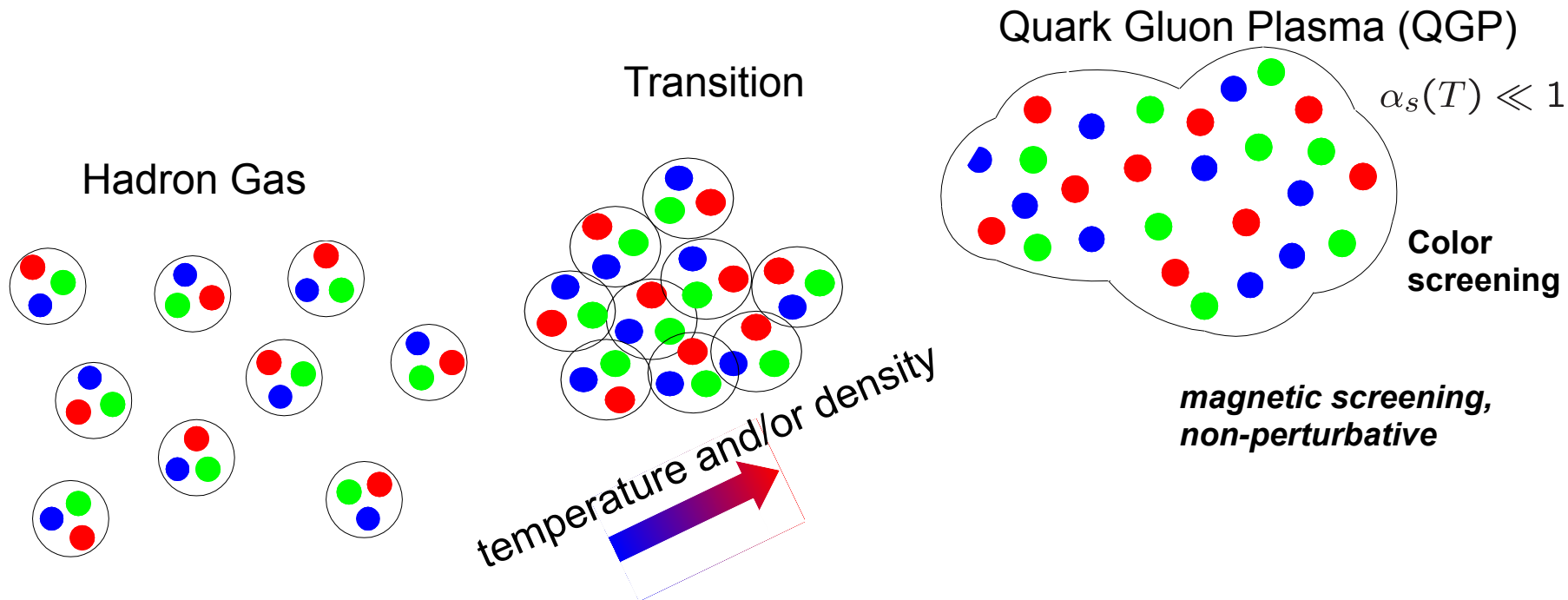
“What does QCD predict for the properties of strongly interaction matter ?”



Challenges for LQCD:

- 1) Equation of state
- 2) Phase diagram and the transition temperature
- 3) Fluctuations of conserved charges
- 4) In-medium hadron spectral functions
- 5) Transport coefficients

Deconfinement at high temperature and density

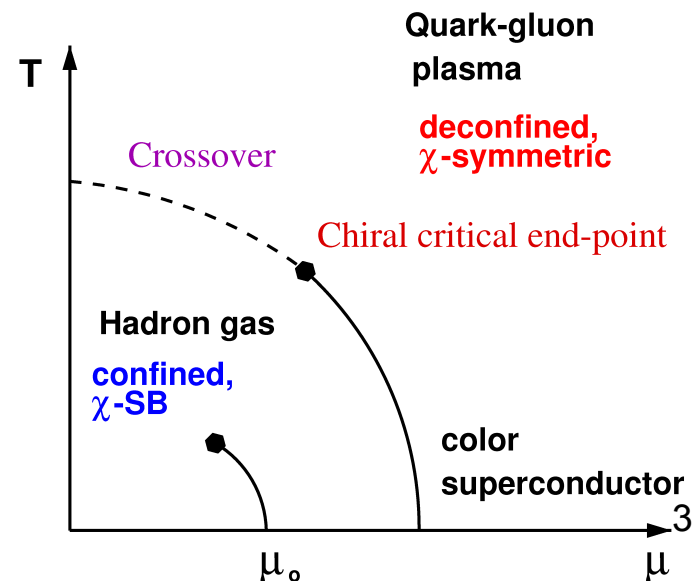


$$T_c = (154 \pm 9) \text{ MeV}$$

Chiral symmetry is broken in the low temperature (hadronic) “phase”

but is restored at high T

QCD analog of
ferromagnet-paramagnet transition



Physics of heavy ion collisions and LQCD

high temperature QCD
weak coupling ?

Chiral transition, T_c fluctuations of conserved charges

EoS,
viscosity

Initial State:
colliding nuclei

Quark Gluon Plasma &
hydrodynamic expansion

hadronic rescattering
& freeze-out

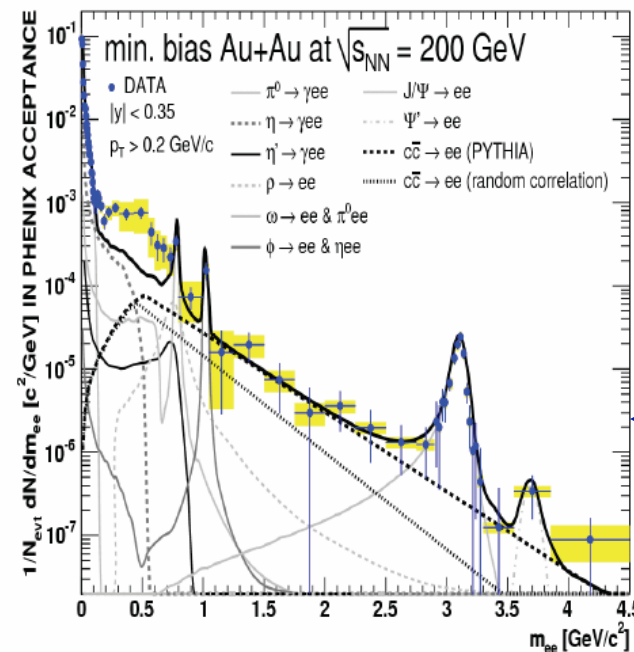
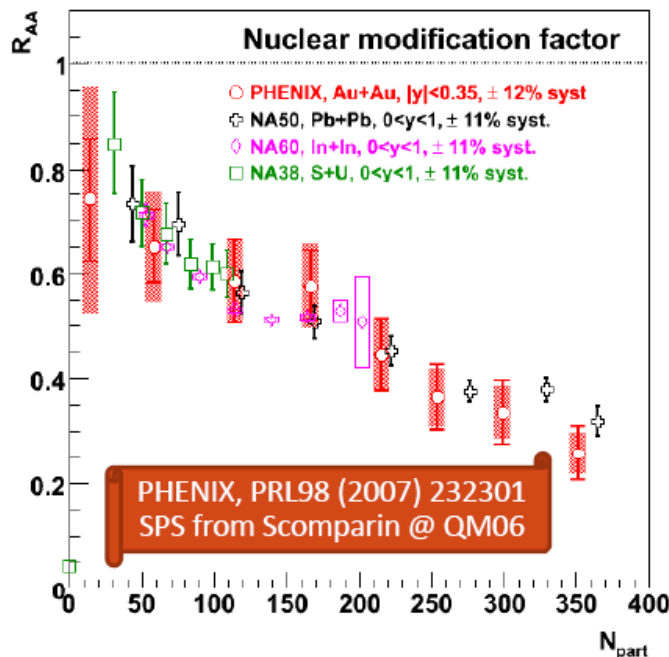
Equilibration:
turbulent color fields

EM and heavy
flavor probes

Hadronization

test of Hadron
Resonance Gas
(HRG)
using LQCD

quarkonium spectral
functions,
heavy quark diffusion,
thermal dileptons



Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \text{Tr} O e^{-\beta H - \mu N} \quad \beta = 1/T$$



$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0, \mathbf{x}) = A_\mu(\beta, \mathbf{x}) \quad \psi(0, \mathbf{x}) = -\psi(\beta, \mathbf{x})$$

Lattice

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(\det D_q[U, m, \mu]) e^{-\sum_x S_G[U(x)]}, U_\mu(x) = e^{igaA_\mu(x)}$$

Generation of gauge configuration (Markov chain) $\mu = 0$ ➡ Hybrid Monte-Carlo

Most of cycles : $D_q^{-1} A$

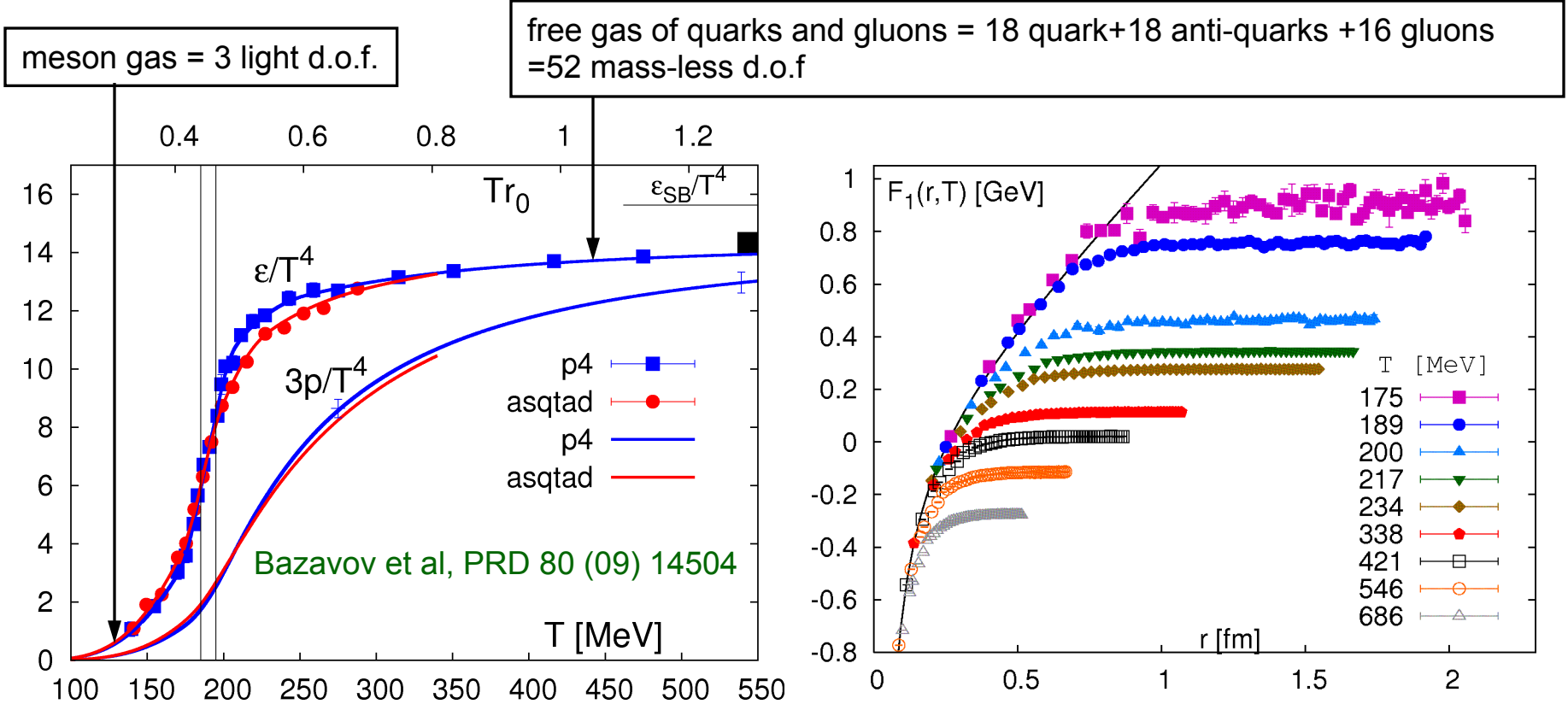
$\mu \neq 0$: $\det D_q(U, m, \mu)$ complex ➡ sign problem ➡ Taylor expansion for not too large μ

Most of cycles : $D_q^{-1} A$

- Highly Improved Staggered Quark (HISQ) relatively inexpensive numerical but does not preserve all the symmetries of QCD (except for zero a)
- Domain Wall Fermion (DWF) formulation: preserves all the symmetries but costs $\sim 100x$ of staggered formulation

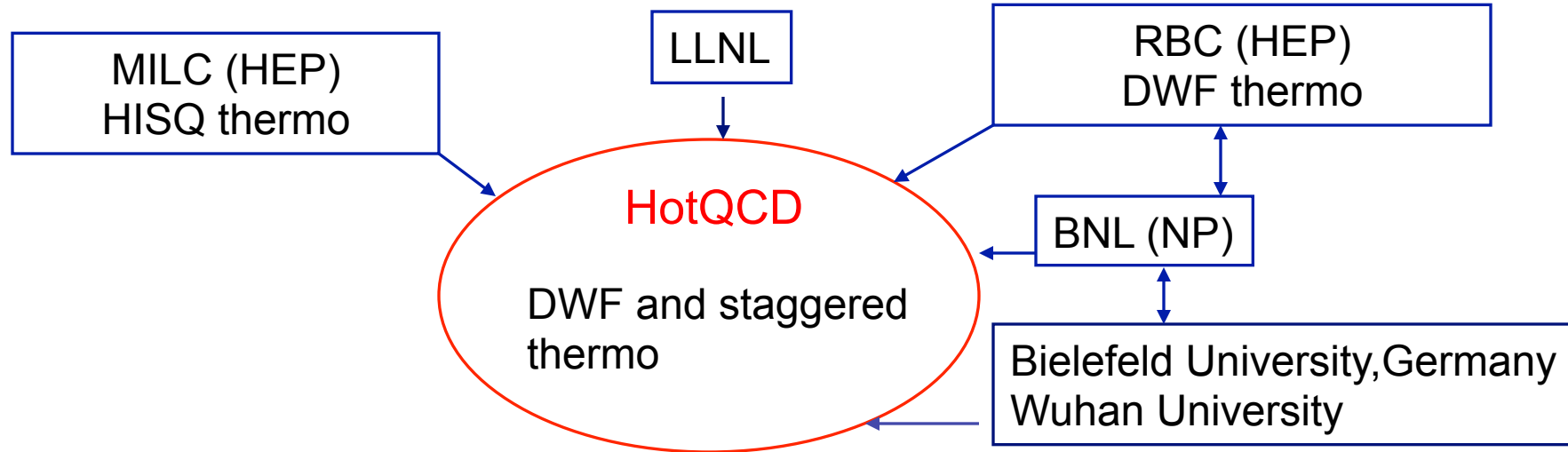
Need to take the limit of zero a : cost $\sim 1/a^7$, $T=1/(a N_\tau)$

Deconfinement : pressure, energy density and color screening



- rapid change in the number of degrees of freedom at $T=160-200\text{MeV}$: **deconfinement**
- deviation from ideal gas limit is about **10%** at high T consistent with the weakly coupled quark gluon gas
- free energy of static quark anti-quark pair shows Debye screening \Rightarrow quarkonium suppression @RHIC

HotQCD : a collaborative effort



Software (USQCD):

MILC code (MIMD C code + platform dependent optimization at a lower level) also for GPU (CUDA)

Columbia Physics System, CPS (C++, optimized for BG/Q, DWF only)

Optimized CUDA GPU code from Bielefeld U.

Resource in 2013-2014 (in core h):

- 1) CPU clusters of USQCD: 40M
- 2) GPU clusters of USQCD: 2.5 M
- 3) INCITE allocation of USQCD: 120M (Mira) 44M (Titan, projected)
- 4) BNL, BG/Q: 18M, BG/L: 50M,
- 5) GPU cluster at LLNL and Bielefeld U.: 3.1M
- 6) BG/Q in Europe: 15M
- 7) **NERSC allocation (PI Bazavov): 10M**
- 8) **BG/Q (LLNL) : 100 M (DWF thermo only)**

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

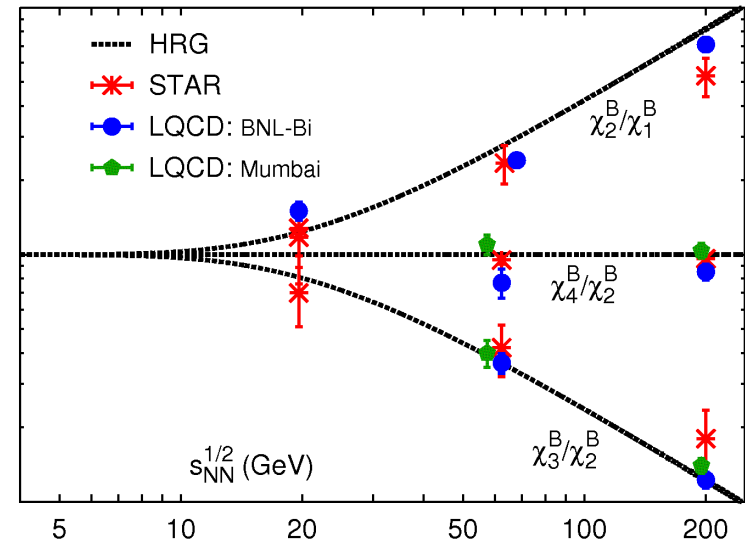
$$\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$

LQCD : Taylor expansion coefficients \Rightarrow fluctuations of conserved charges: $X = B, S, Q$ \Rightarrow Beam energy scan @ RHIC \rightarrow parameters of the distribution

$\chi_1^X = \frac{1}{VT^3} \langle N_X \rangle$	$M_X = \langle N_X \rangle$	mean
$\chi_2^X = \frac{1}{VT^3} \langle (\delta N_X)^2 \rangle$	$\sigma_X = \langle (\delta N_X)^2 \rangle$	variance
$\chi_3^X = \frac{1}{VT^3} \langle (\delta N_X)^3 \rangle$	$S_X = \langle (\delta N_X)^3 \rangle / \sigma_X^3$	Skewness
$\chi_4^X = \frac{1}{VT^3} [\langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X)^2 \rangle^2]$	$K_X = \langle (\delta N_X)^4 \rangle / \sigma_X^4 - 3$	Kurtosis

$$N_X = X - \bar{X}, \quad \delta N_X = N_X - \langle N_X \rangle$$

can be calculated very effectively on single GPUs



Volume independent combinations:

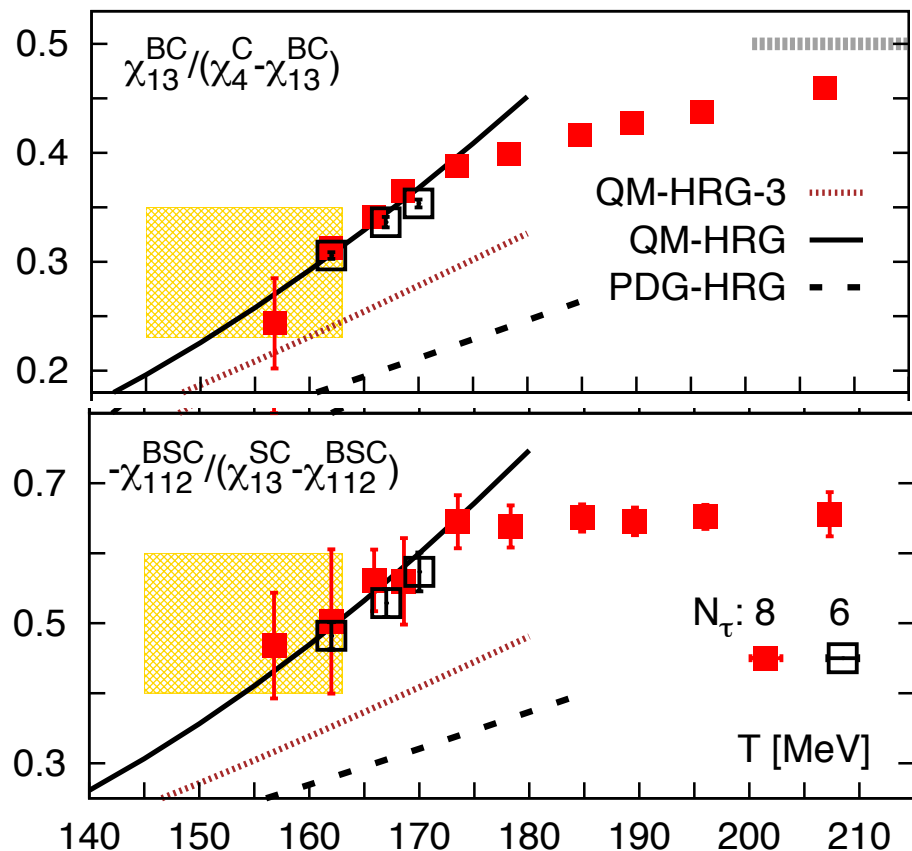
$$M_X / \sigma_X = \chi_1^X / \chi_2^X$$

$$S_X \cdot \sigma_X = \chi_3^X / \chi_2^X$$

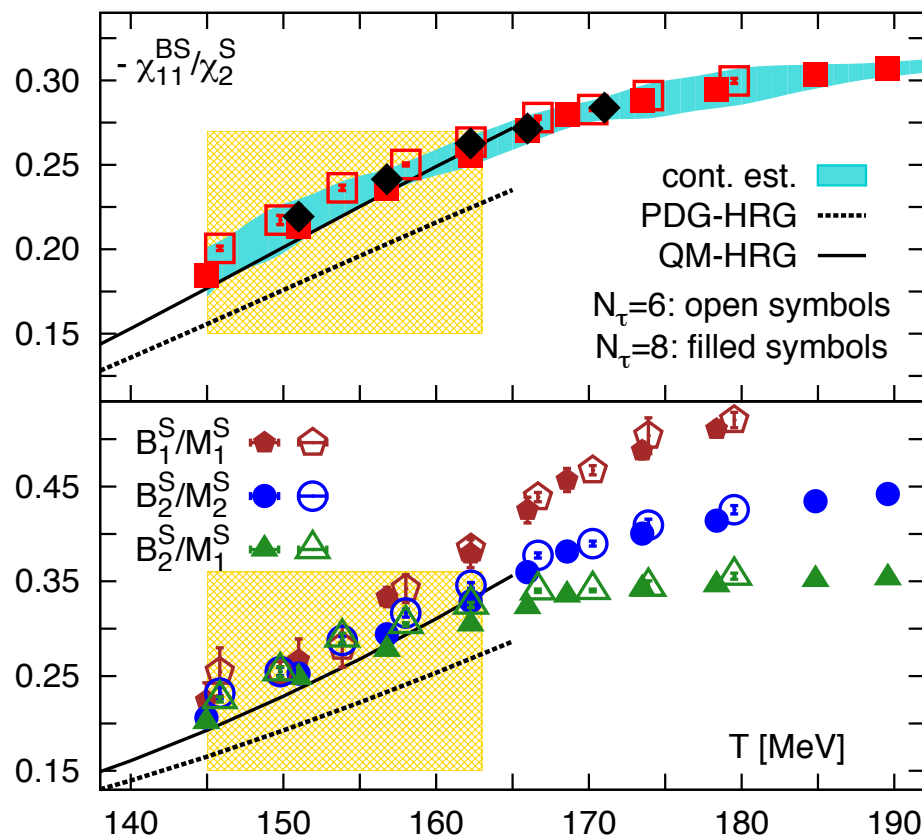
$$K_X \cdot \sigma_X^2 = \chi_4^X / \chi_2^X$$

Charge and strangeness fluctuations and missing hadrons

Bazavov et al, arXiv:1404.4043v1



Bazavov et al, arXiv:1404.6511

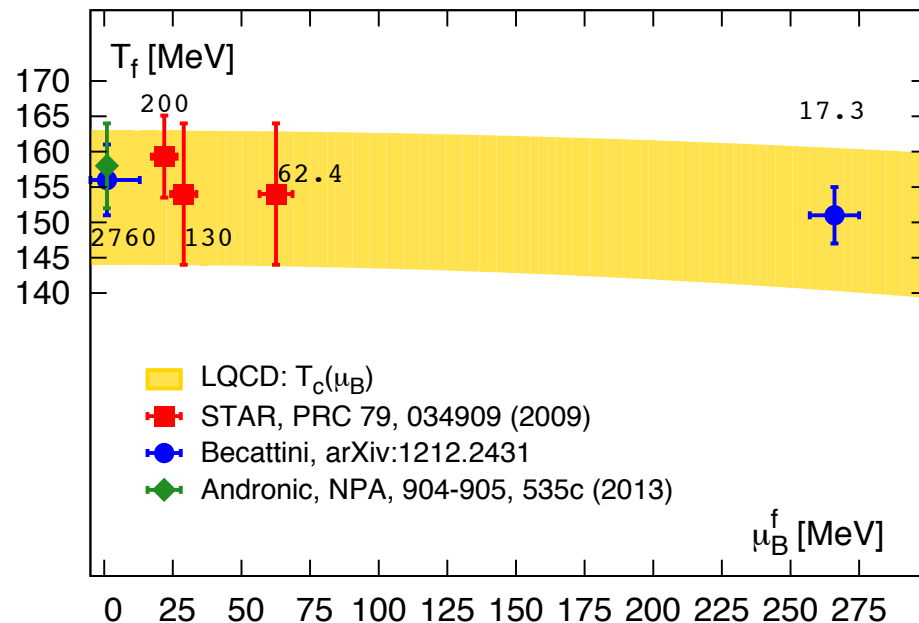
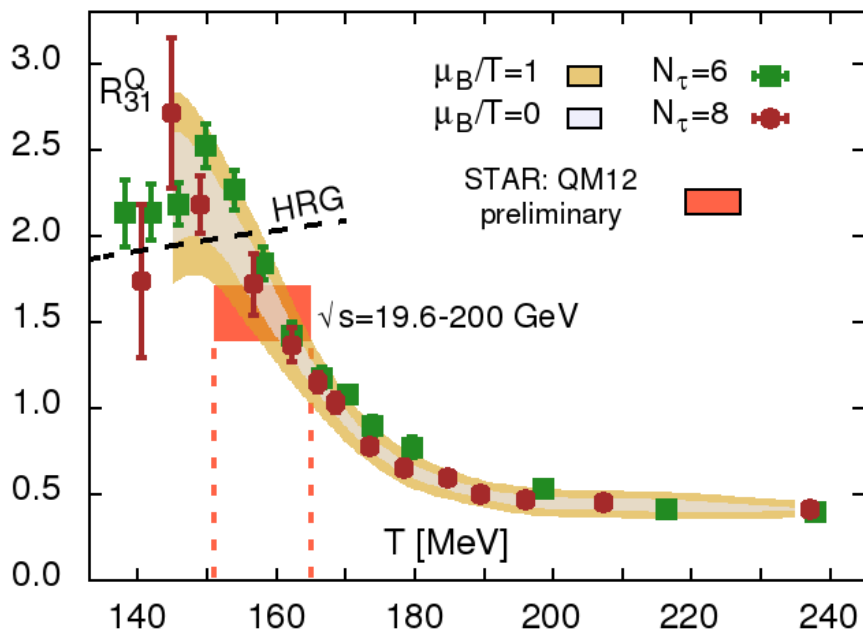


Experimental knowledge of strange and charm hadron spectrum is rather incomplete
Future experiments @ Jlab and FAIR (Germany) will address this problem

HRG that includes hadron states predicted by by quark model (also LQCD)
agrees better with lattice results that HRG with PDG states only !

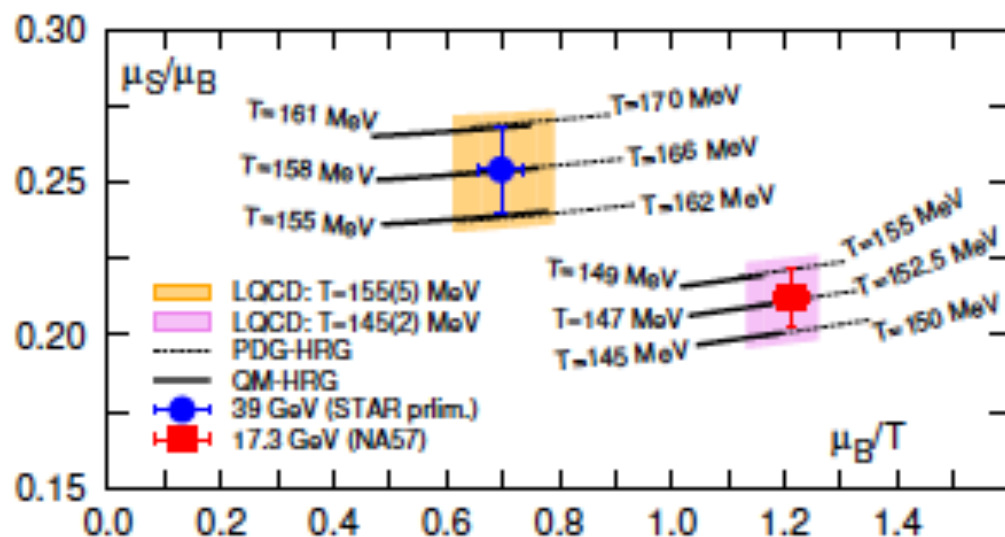
QCD thermodynamics at non-zero chemical potential

Bazavov et al, PRL 109 (2012) 192302, Mukherjee, Wagner, arXiv:1307.6255



For consistent description of
The freeze-out of strange hadrons
Need to include the contribution of
“missing states”

Bazavov et al, arXiv:1404.6511



Spectral functions at $T>0$ and physical observables

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Heavy meson spectral functions:



quarkonia properties at $T>0$
heavy quark diffusion in QGP: D

$$J_H = \bar{\psi} \Gamma_H \psi$$

Heavy flavor probes at RHIC

Light vector meson spectral functions:

$$J_\mu = \bar{\psi} \gamma_\mu \psi$$



thermal dilepton production rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2} \frac{1}{e^{\omega/T} - 1} \frac{\sigma_{\mu\mu}(\omega, p, T)}{\omega^2 - p^2}$$

thermal photon production rate :

$$p \frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi} \frac{1}{e^{p/T} - 1} \sigma_{\mu\mu}(\omega = p, p, T)$$

Thermal photons and dileptons provide information about the temperature of the

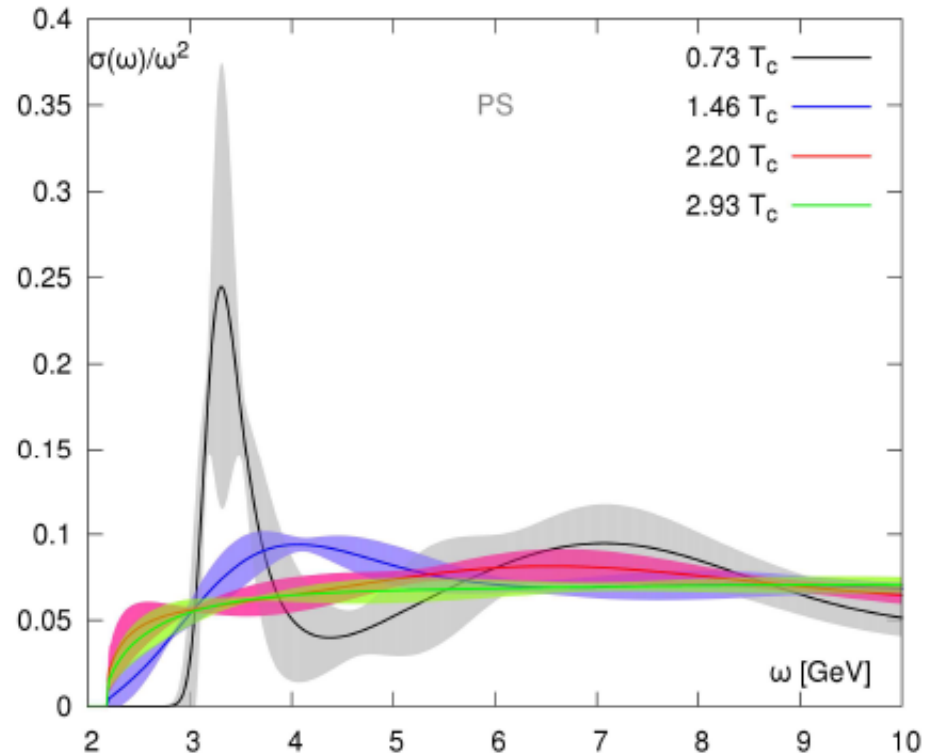
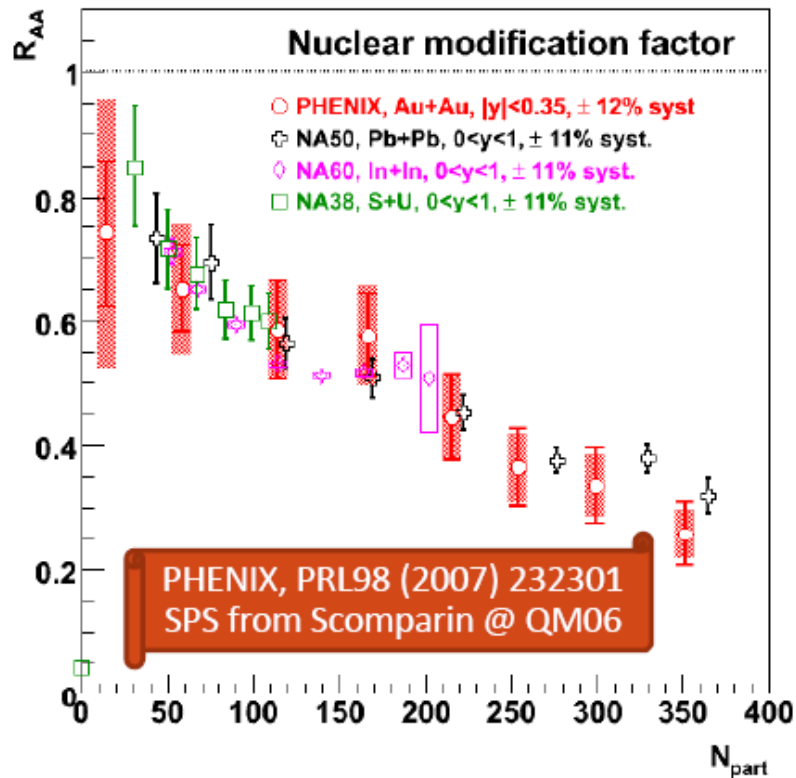
electric conductivity ζ :

Quarkonium spectral spectral functions

Charmonium spectral functions on isotropic lattice in quenched approximation with Wilson quarks:

H.-T. Ding et al, arXiv:1204.4945

$N_\tau=24-96$, $a^{-1}=18.97\text{GeV}$



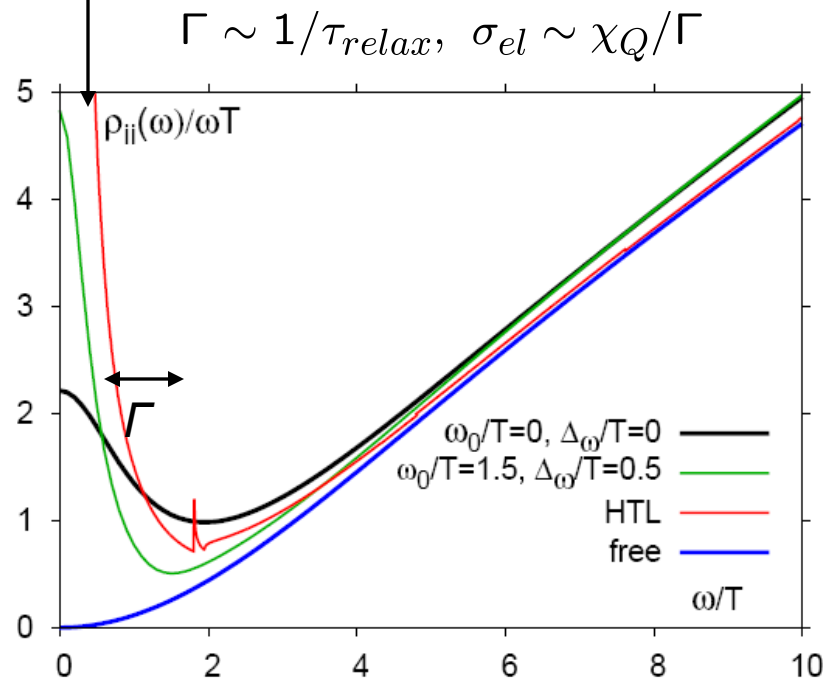
No clear evidence for charmonium bound state peaks above T_c in spectral functions !

Lattice calculations of transport coefficients

Electric conductivity:

Ding et al, PRD 83 (11) 034504

peak at $\omega \approx 0$ = transport peak



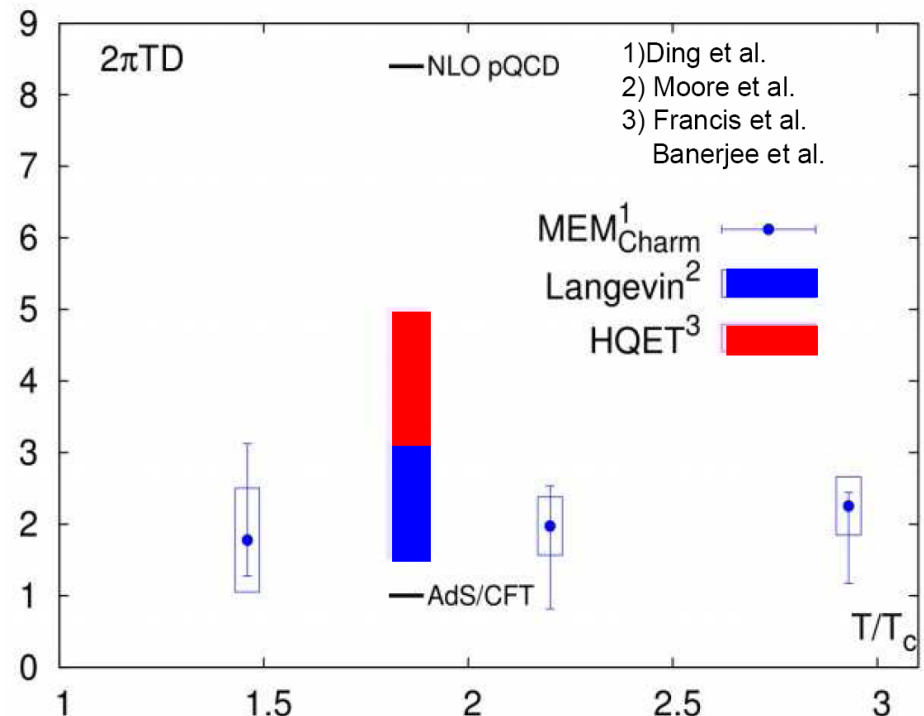
$$1/3 < \frac{1}{C_{em}} \frac{\sigma_{el}}{T} < 1, C_{em} = \sum_f Q_f^2$$

Heavy quark diffusion constant:

Ding et al, arXiv:1204:4954

Banarjee et al, arXiv:1109.5738

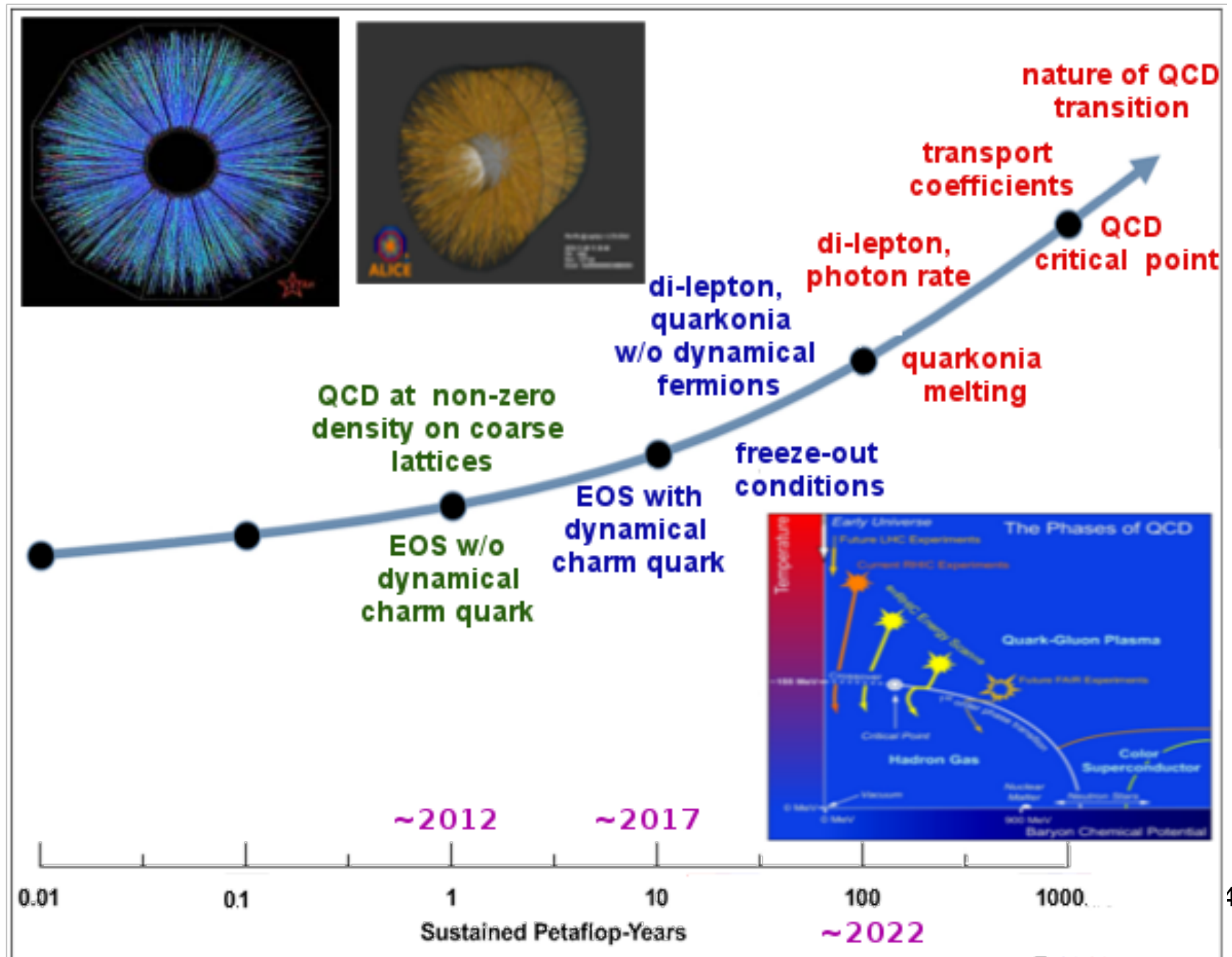
Kaczmarek et al, arXiv:1109:3941



Quenched QCD calculations up to $128^3 \times 32$ lattice

Perfect liquid: $\eta/s = 1/(4\pi) \Leftrightarrow$ no transport peak !

Summary



Back-up slide

